Understanding Spatial Regression Models from a Weighting Perspective in an Observational Study of the Impact of Superfund Cleanups on Birth Weight

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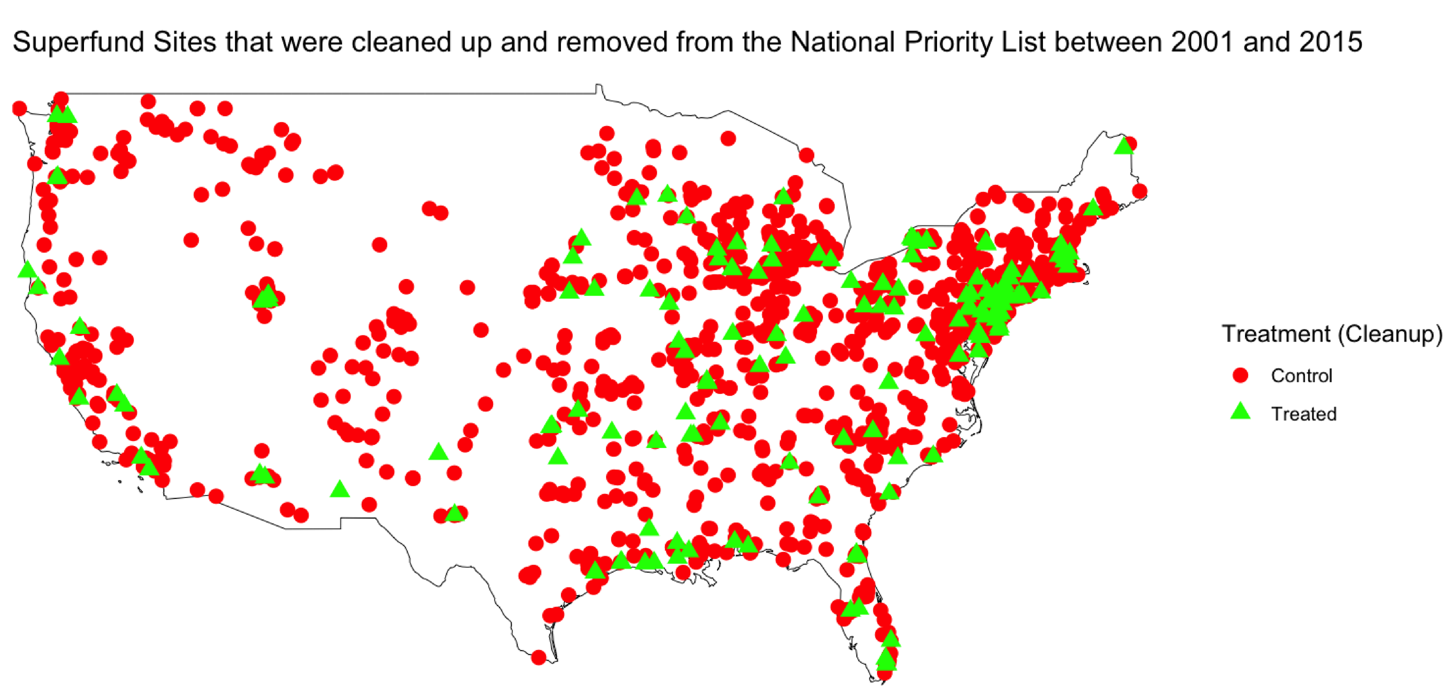
Superfund cleanups and birth weight

More than 78 million Americans — 24% of the U.S. population, including 17 million children and 11 million elderly individuals — live within three miles of a hazardous waste site [1]. In 1980, Congress enacted the Comprehensive Environmental Response, Compensation, and Liability Act, granting the Environmental Protection Agency authority to identify and remediate these sites, which became known as **Superfund sites**. Remediation remains **contentious** due to high costs — often tens of millions per site — and disputes over whether the benefits justify the expense [2].

Q: Does Superfund cleanup impact the proportion of low birth weight births?

- **Binary treatment**: $Z_i \in \{0,1\}$ indicates whether the ith Superfund site was remediated between 2001 and 2015.
- **Measured confounders**: $\boldsymbol{X}_i \in \mathbb{R}^{11}$ denotes a vector of 10 sociodemographic confounders derived from the 2000 Decennial Census, measured within 2 km buffers around each site, combined with an intercept.
- **Outcome**: $Y_i \in \mathbb{R}$ is the proportion of low birth weight (birth weight ≤ 2500 grams) births within a 2 km buffer around site i during the period 2016 to 2020.



One key challenge: unmeasured spatial confounding. Unmeasured spatial confounders are unobserved spatially structured variables that influence both treatment assignment and outcomes, potentially leading to biased effect estimates and invalid confidence intervals [3, 4]. **Examples:** community engagement and local funding availability.

Spatial regression is standard practice

To account for potential unmeasured spatial confounding, researchers frequently introduce a mean zero spatially autocorrelated error term into a linear regression model and interpret the treatment coefficient as a causal effect. The general form of a spatial regression model is

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \tau \boldsymbol{Z} + \boldsymbol{\epsilon},$$
$$\boldsymbol{\epsilon} \perp\!\!\!\perp (\boldsymbol{X}, \boldsymbol{Z}),$$
$$\mathbb{E}(\boldsymbol{\epsilon}) = \boldsymbol{0}, \mathrm{Var}(\boldsymbol{\epsilon}) = \boldsymbol{\Sigma} = \sigma^2 \boldsymbol{I}_n + \rho^2 \boldsymbol{S}$$

where the covariance matrix of the errors, $\boldsymbol{\Sigma}$, is comprised of both independent and identically distributed error ($\sigma^2 \boldsymbol{I}_n$) and spatially autocorrelated error ($\rho^2 \boldsymbol{S}$).

The generalized least squares (GLS) estimator of τ is

$$\begin{pmatrix} \hat{\beta}_{GLS} \\ \hat{\tau}_{GLS} \end{pmatrix} = \left((\boldsymbol{X} \ \boldsymbol{Z})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{X} \ \boldsymbol{Z}) \right)^{-1} (\boldsymbol{X} \ \boldsymbol{Z})^T \boldsymbol{\Sigma}^{-1} \boldsymbol{Y}. \quad (1)$$

Three common examples

1. For a model with random intercepts at the state-level, $\boldsymbol{S}$ is a block diagonal matrix with kth block equal to J_{n_k}, where $n_k = \sum_{i=1}^n I(C_i = k)$ and J_{n_k} is a $n_k \times n_k$ matrix of ones.
2. For an intrinsic conditional autoregressive model, $\boldsymbol{S} = (\boldsymbol{D} - \boldsymbol{W})^{-1}$ where $\boldsymbol{W}$ is a spatial weighting matrix and $\boldsymbol{D}$ is the diagonal matrix with ith entry equal to $\sum_{j=1}^n W_{ij}$.
3. For a Gaussian process model, $\boldsymbol{S} = \boldsymbol{K}$ where $\boldsymbol{K}$ is the kernel of the Gaussian process.

Q: Can spatial regression adjust for unmeasured spatial confounding?

The prevailing view is that spatial regression **should not be used** to adjust for unmeasured spatial confounding, due to the bias of the treatment coefficient estimate in finite samples. Recently, [5] established that spatial regression models with Gaussian process covariance yield **consistent** treatment effect estimates under unmeasured confounding that is continuous in space.

Spatial regression is designed to approximately balance the means of a hidden set of spatial covariates, thereby adjusting for a specific class of unmeasured confounders with spatial autocorrelation.

An encompassing weighting framework

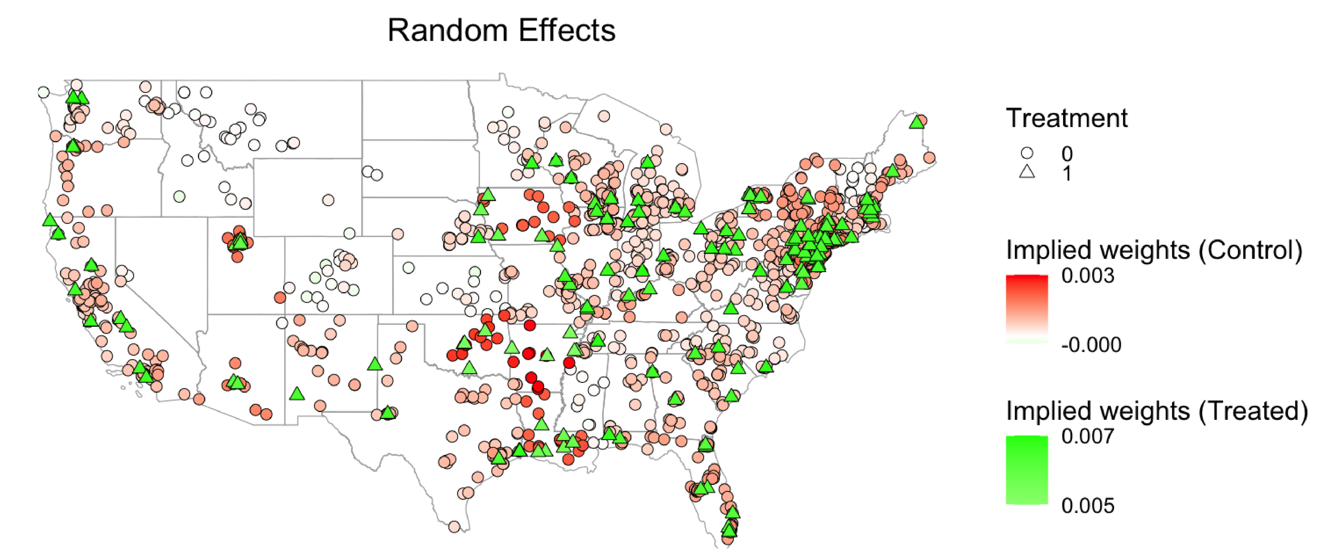
Proposition 1 (GLS estimators are weighting estimators) The GLS estimator of τ in Equation 1 can be expressed as

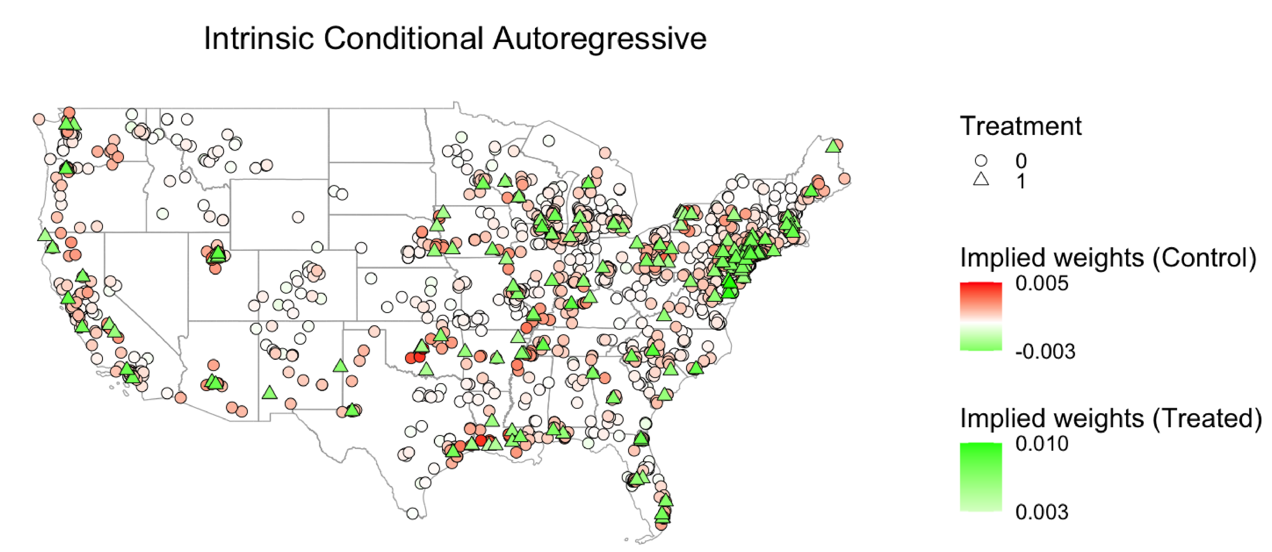
$$\hat{\tau}_{GLS} = \sum_{i:Z_i=1} w_i Y_i - \sum_{i:Z_i=0} w_i Y_i$$

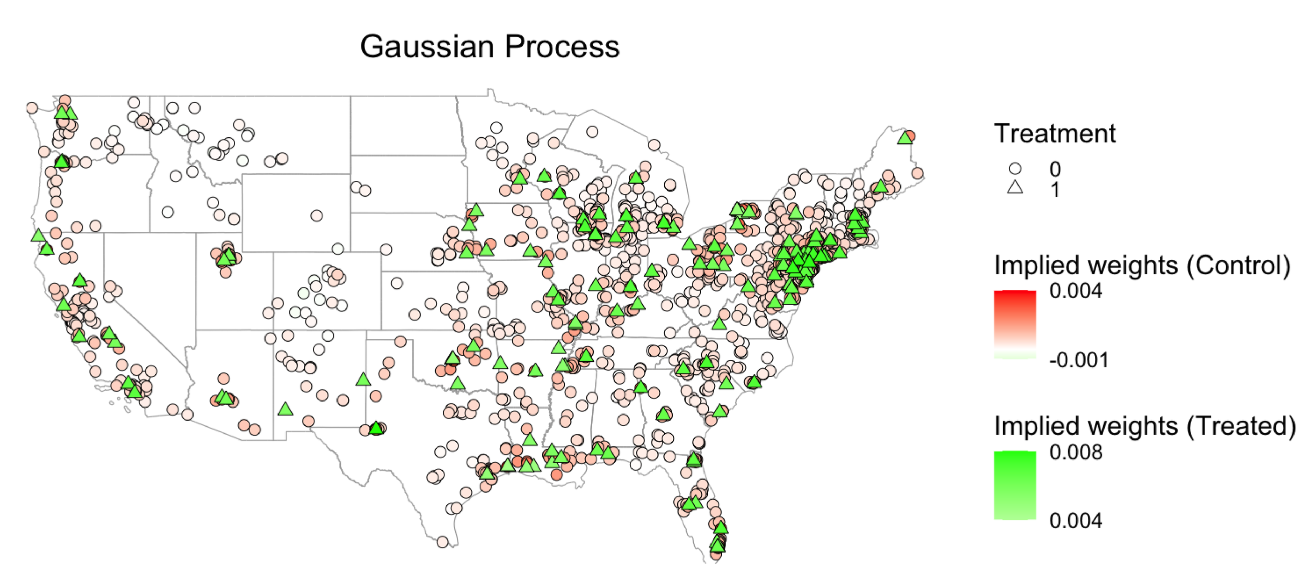
for some weights of $w_1, \ldots, w_n$. A closed-form expression for the weights are

$$(w_1, \ldots, w_n)^T = \boldsymbol{M} \frac{(\boldsymbol{I}_n - \boldsymbol{\Sigma}^{-1}\boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T)\boldsymbol{\Sigma}^{-1}\boldsymbol{Z}}{\boldsymbol{Z}^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{I}_n - \boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1})\boldsymbol{Z}}$$

where $\boldsymbol{M}$ is the diagonal matrix with (i,i) entry $M_{ii} = 2Z_i - 1$.







Proposition 2 (GLS estimators are minimal dispersion balancing weights estimators) Let $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ be the eigenvectors of $\boldsymbol{S}$ with corresponding eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n \geq 0$. Consider the following quadratic programming problem:

$$\min_{\boldsymbol{w}} \left\{ \sigma^2 \sum_{i=1}^n w_i^2 + \rho^2 \sum_{k=1}^n \lambda_k \left(\sum_{i:Z_i=1} w_i v_{ki} - \sum_{i:Z_i=0} w_i v_{ki} \right)^2 \right\} \quad (2)$$

$$\text{subject to} \begin{cases} \sum_{i:Z_i=1} w_i = 1, \sum_{i:Z_i=0} w_i = 1 \\ \sum_{i:Z_i=1} w_i \boldsymbol{X}_i = \sum_{i:Z_i=0} w_i \boldsymbol{X}_i \end{cases} \quad (3)$$

A solution to this problem are the implied weights of the GLS estimator,

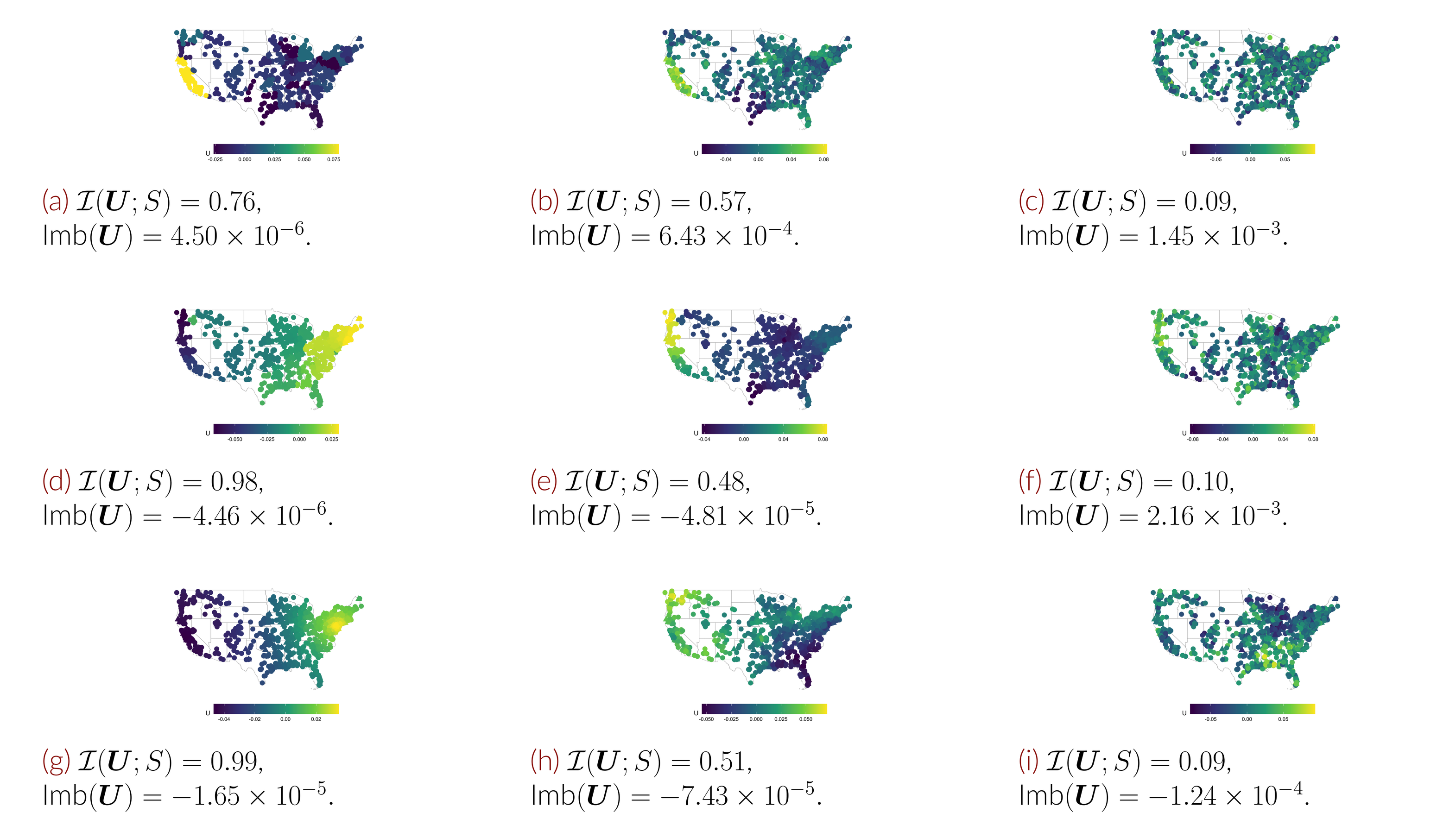
$$\boldsymbol{w} = \boldsymbol{M} \frac{(\boldsymbol{I}_n - \boldsymbol{\Sigma}^{-1}\boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T)\boldsymbol{\Sigma}^{-1}\boldsymbol{Z}}{\boldsymbol{Z}^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{I}_n - \boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1})\boldsymbol{Z}}.$$

Unmeasured confounding bias and Moran's I

Proposition 3 Suppose that consistency, positivity, and conditional ignorability given (X, U) hold. Further suppose the outcome model is linear, i.e. $Y_i = \boldsymbol{\beta}^T X_i + \tau Z_i + \gamma U_i + \epsilon_i$. Then the bias of the spatial regression estimator $\hat{\tau}_{GLS}$ in estimating $\tau_{\text{ATT}} = \tau$, conditional on $(\boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{U})$, is:

$$\left| \mathbb{E}(\hat{\tau}_{GLS} | \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{U}) - \tau \right| = \left| \gamma \Big(\sum_{i:Z_i=1} w_i U_i - \sum_{i:Z_i=0} w_i U_i \Big) \right|$$
$$\leq |\gamma| \sqrt{ \frac{c_0 (2\sigma^2 + \rho^2\lambda_1 + \rho^2\lambda_n)^2}{4(\sigma^2 + \rho^2\lambda_1)(\sigma^2 + \rho^2\lambda_n)(\sigma^2 + \rho^2\lambda_1 \mathcal{I}(\boldsymbol{U}; S))} \sum_{i=1}^n (U_i - \bar{U})^2 }$$

where $c_0 = \sigma^2 \sum_{i=1}^n w_i^2 + \rho^2 \sum_{k=1}^n \lambda_k (\sum_{i:Z_i=1} w_i v_{ki} - \sum_{i:Z_i=0} w_i v_{ki})^2$ and $\mathcal{I}(\boldsymbol{U}; S)$ is the Moran's I statistic of $\boldsymbol{U}$, a widely used measure of spatial autocorrelation.



(a) $\mathcal{I}(\boldsymbol{U}; S) = 0.76$, $\mathrm{Imb}(\boldsymbol{U}) = 4.50 \times 10^{-6}$.
(b) $\mathcal{I}(\boldsymbol{U}; S) = 0.57$, $\mathrm{Imb}(\boldsymbol{U}) = 6.43 \times 10^{-4}$.
(c) $\mathcal{I}(\boldsymbol{U}; S) = 0.09$, $\mathrm{Imb}(\boldsymbol{U}) = 1.45 \times 10^{-3}$.
(d) $\mathcal{I}(\boldsymbol{U}; S) = 0.98$, $\mathrm{Imb}(\boldsymbol{U}) = -4.46 \times 10^{-6}$.
(e) $\mathcal{I}(\boldsymbol{U}; S) = 0.48$, $\mathrm{Imb}(\boldsymbol{U}) = -4.81 \times 10^{-5}$.
(f) $\mathcal{I}(\boldsymbol{U}; S) = 0.10$, $\mathrm{Imb}(\boldsymbol{U}) = 2.16 \times 10^{-3}$.
(g) $\mathcal{I}(\boldsymbol{U}; S) = 0.99$, $\mathrm{Imb}(\boldsymbol{U}) = -1.65 \times 10^{-5}$.
(h) $\mathcal{I}(\boldsymbol{U}; S) = 0.51$, $\mathrm{Imb}(\boldsymbol{U}) = -7.43 \times 10^{-5}$.
(i) $\mathcal{I}(\boldsymbol{U}; S) = 0.09$, $\mathrm{Imb}(\boldsymbol{U}) = -1.24 \times 10^{-4}$.

Figure 1. Three examples of unmeasured confounders with Moran's I and their mean imbalances. **Top row** shows three examples from the Random Effects model, **middle row** shows three examples from the intrinsic conditional autoregressive model, and **bottom row** shows three examples from the Gaussian Process model.

A more general spatial weighting approach

We propose an alternative estimator that accounts for **diverse forms** of unmeasured spatial confounding while **relaxing the assumptions of linearity and effect homogeneity** that spatial regression relies on.

Let $X^{\text{aug}} = (\boldsymbol{X}, \boldsymbol{V}) \in \mathbb{R}^{n \times (11+J)}$ be the set of measured covariates including intercept, augmented with J pre-selected eigenvectors $\boldsymbol{V} = (\boldsymbol{v}_1, \ldots, \boldsymbol{v}_J) \in \mathbb{R}^{n \times J}$. Building on [6, 7, 8], consider the following quadratic programming problem in the control group:

$$\min_{\boldsymbol{w}} \sum_{i:Z_i=0} w_i^2, \text{ subject to} \begin{cases} \left| \sum_{i:Z_i=0} w_i B_k(\boldsymbol{X}_i^{\text{aug}}) - \frac{1}{n_t} \sum_{i:Z_i=1} B_k(\boldsymbol{X}_i^{\text{aug}}) \right| \leq \delta_k, k = 1, 2, \ldots, K \\ \sum_{i:Z_i=0} w_i = 1, w_i \geq 0 \ \forall i \end{cases}$$

where $B_k(\boldsymbol{X}^{\text{aug}}), k = 1, 2, \ldots, K$ is a set of basis functions and $n_t = \sum_{i=1}^n Z_i$.

Define the spatial weighting estimator of the ATT as $\hat{\tau}_{SW} = \frac{1}{n_t} \sum_{i:Z_i=1} Y_i - \sum_{i:Z_i=0} w_i Y_i$ where $\{w_i\}_{Z_i=0}$ are the weights obtained by solving the optimization problem above.

Cleanup may reduce the proportion of low birth weight births

Method	Risk Ratio Estimate
Ordinary least squares	0.95 (0.88, 1.02)
Random effects	0.94 (0.84, 1.07)
Conditional autoregressive	0.92 (0.83, 1.04)
Gaussian process	0.94 (0.84, 1.06)
Spatial weighting	0.93 (0.85, 1.02)

Estimates of $\mathbb{E}(Y(1)|Z=1)/\mathbb{E}(Y(0)|Z=1)$ and 95% confidence intervals from five methods, assessing the impact of Superfund cleanups (2001–2015) on the proportion of low birth weight births (2016–2020) within 2 km buffers around sites in New Jersey and Pennsylvania.

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